行政院國家科學委員會專題研究計畫成果報告

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1. 摘要

在本研究報告中，我們在擬 H-空間及積擬 H-空間上，使用全域交集定理，建立了集合值映射的定點存在定理。並將此結果應用在抽象經濟上，建立了平衡點的存在理論。最後，我們也在擬 H-空間上，導出了樊璣型不等式成立的結果。

關鍵詞：擬 H-空間, q-映射, 積空間, 定點定理, 平衡點, 抽象經濟, Ky Fan 型不等式.

Abstract

In this projection report, we construct some fixed point results for set-valued maps on pseudo H-spaces and product spaces of pseudo H-spaces by using the whole intersection theorem (Theorem 1 in section 3). As applications, we establish the existence result of the equilibrium point in the abstract economies. We also derive a Ky Fan Type minimax inequality on pseudo H-spaces.

Keywords: Pseudo H-spaces, q-maps, Product Space, Fixed Point Theorem, Equilibrium Point, Abstract Economy, Ky Fan type inequality.

2. Prelimilies

Let I be an index set and for each c, let E_i be a topological space. Let $E=\prod_{i\in I} E_i$. For each $i\in I$, let $T_i : E \to 2^{E_i}$ be a set-valued map. A point $\bar{x}\in E$ is called a fixed point of $T(x)=\prod_{i\in I} T_i(x)$ if $\bar{x}\in T(\bar{x})$. That is, $\bar{x}_i \in T_i(\bar{x})$ for each $i\in I$, where $\bar{x}_i$ is the projection of $\bar{x}$ onto E_i.

In 1992, E. Tarafdar[T92] established the model of the abstract economies in product H-spaces [H87], and discussed the existence of equilibrium points of the abstract economies by using the fixed point theorem on the product H-spaces. E. Marchi et al [ML93] used the Peleg's theorems to establish the fixed point theorem in the product H-spaces, and deduce the Ky Fan type inequalities and present a generalization of Ky Fan's intersection theorem for sets with convex section.

In the recent years, the fixed point theorems on the product space of various generalized spaces have been discussed by many authors. Lin et al [LP98] established the theorems on G-convex spaces. Ansari el al [ALY00] discussed the theorem on the product space of Hausdorff topological spaces with applications to abstract economies. D. O'Regan [O98] discussed the theorem on Hausdorff topological space by using the DKT mapping, and also discussed the existence theorem of equilibrium point in abstract economies.

The main results of this projection report is to derive more general fixed point theorems on the product space of pseudo H-spaces. From this direction, we can establish the existence theorems of equilibrium point of abstract economies.

Now, we first define the pseudo H-space and q-map as follows.

Definition 1. Let X be a topological space. The pair (X, q) is said to be a pseudo H-space if for each nonempty finite subset A of X, the mapping q : $\triangle^{|A|-1} \to 2^X$ is continuous with nonempty compact values.

Lemma 1. Any finite product space of pseudo H-spaces is also a pseudo H-space.

Definition 2. Let (X, q) be a pseudo H-space. A mapping F : X $\to 2^X$ is a q-map if For each nonempty finite subset A of X, $q(\triangle^{|A|-1}) \subset \bigcup_{x\in A} F(x)$ and $q(\triangle^{|J|-1}) \subset \bigcup_{x\in J} F(x)$ for all nonempty finite subset J of A.

3. An Intersection Theorem.

We first discuss the following intersection theorem.

Theorem 1. Let (X, q) be a pseudo H-space. A mapping F : X $\to 2^X$ is a q-map with compactly closed (or compactly open) values. Then $\bigcap_{x\in L} F(x) \neq \emptyset$ for all nonempty finite subset L of X. Furthermore, if $\bigcap_{x\in N} \overline{F(x)}$ is compact for some nonempty finite subset N of X, then $\bigcap_{x\in X} \overline{F(x)} \neq \emptyset$.

4. Some Fixed Point Results.

We shall use Theorem 1 to derive the following fixed point theorems.

Theorem 2. Let (X, q) be a pseudo H-space. The mapping O : X $\to 2^X$ has open values. f : X $\to 2^X$. Suppose that

(1) for each y∈ X, for each nonempty finite subset A of $X\backslash(O^c)^{-1}(y)$, $q(\triangle^{|A|-1}) \subset X\backslash(O^c)^{-1}(y)$,

(2) f^{-1} (x) contains at least one O(x) for each x∈ X,

(3) $X= \bigcup_{x\in X} O(x)$; and

(4) $\bigcap_{x\in N} O^c(x)$ is compact for some nonempty finite subset N of X.

Then f has fixed point.

The condition (1) in Theorem 2 can be replace by (1') as follows, the conclusion still hold.

Theorem 3. Let (X, q) be a pseudo H-space. The mapping O : X $\to 2^X$ has open values. f : X $\to 2^X$. Suppose that

(1') for each w∈ X, for each nonempty finite subset A of f(w), $q(\triangle^{|A|-1}) \subset f(w)$,

(2) f^{-1} (x) contains at least one O(x) for each x∈ X,

(3) $X= \bigcup_{x\in X} O(x)$; and

(4) $\bigcap_{x\in N} O^c(x)$ is compact for some nonempty finite subset N of X.

Then f has fixed point.

By using Theorem 3, we can discuss the following fixed point theorem in the product space of pseudo H-spaces.

Theorem 4. Let I be a finite index. For each i∈ I, (X_i, q_i) is a pseudo H-space. $X=\prod_{i\in I} X_i$. $O_i : X_i \to 2^X$ has open values. $f_i : X \to 2^{X_i}$. Suppose that

(1) for each i ∈ I and w ∈ X, for each nonempty finite subset A_i of $f_i(w)$, $q_i(\triangle^{|A_i|-1}) \subset f_i(w)$,

(2) for each i∈ I , $f_i^{-1}(x_i)$ contains at least one $O_i(x_i)$ for each $x_i \in X_i$,

(3) for each $i \in I$, $X = \bigcup_{x_i \in X_i} O_i(x_i)$; and

(4) for each $i \in I$, $\bigcap_{x_i \in N_i} O_i^c(x_i)$ is compact for some nonempty finite subset N_i of X_i.

Then $f = \prod_{i \in I} f_i$ has fixed point.

5. Application to Abstract Economies.

The abstract economic model introduced by Tarafdar [T92] is described on H-space. By adapt the technique of Tarafdar, we can discuss the abstract economies on pseudo H-space. Now, we state the model as follows. Let $\{(X_i, q_i) : i \in I\}$ be a family of pseudo H-spaces, where I is an index. For each $i \in I$, T_i: $X = \prod_{i \in I} X_i \rightarrow 2^{X_i}$ is the constraint set-valued mapping and U_i: $X \rightarrow R$ is the utility or pay off function. For each $i \in I$, the preference set-valued mapping $P_i : X \rightarrow 2^{X_i}$ is defined by $P_i(x) = \{y_i \in X_i : U_i(y_i, x_{\hat{i}}) > U_i(x)\}$, where $x_{\hat{i}} \in X_{\hat{i}}$ and $X_{\hat{i}} = \prod_{j \in I, j \neq i} X_j$ for each $i \in I$. An abstract economy is defined by $E = \{(X_i, q_i, T_i, P_i): i \in I\}$. A point $\bar{x} \in X$ is called an equilibrium point or a generalized Nash equilibrium point[N50] of the economy E if $U_i(\bar{x}) = U_i(\bar{x}_i, \bar{x}_{\hat{i}}) = \sup_{z_i \in T_i(\bar{x})} U_i(z_i, \bar{x}_{\hat{i}})$ for each $i \in I$. Hence, an equilibrium point $\bar{x} \in X$ of the economy E is given by $\bar{x}_i \in T_i(\bar{x})$ and $P_i(\bar{x}) \cap T_i(\bar{x}) = ¤$ for each $i \in I$.

Theorem 5. Let I be a finite index. For each $i \in I$, (X_i, q_i) is a pseudo H-space. $X = \prod_{i \in I} X_i$. $O_i : X_i \rightarrow 2^X$ has open values. P_i, $T_i : X \rightarrow 2^{X_i}$. Suppose that

(1) for each $i \in I$ and $w \in X$, for each nonempty finite subset A_i of $T_i(w)$, $q_i(\triangle^{|A_i|-1}) \subset T_i(w)$, and for each nonempty finite subset B_i of $P_i(w)$, $q_i(\triangle^{|B_i|-1}) \subset P_i(w)$,

(2) for each $i \in I$, $T_i^{-1}(x_i) \cap (G_i^c \cup P_i^{-1}(x_i))$ contains at least one $O_i(x_i)$ for each $x_i \in X_i$,

(3) for each $i \in I$, $X = \bigcup_{x_i \in X_i} O_i(x_i)$,

(4) for each $i \in I$, $\bigcap_{x_i \in N_i} O_i^c(x_i)$ is compact for some nonempty finite subset N_i of X_i; and

(5) for each $i \in I$ and for each $x \in X$, $x_i \in P_i(x)$ and $T_i(x) \neq ¤$.

Then the abstract economies $E = \{(X_i, q_i, T_i, P_i): i \in I\}$ has an equilibrium point.

6. The Ky Fan type minimax inequality.

Finally, we derive the Ky Fan type minimax inequality[F72] on pseudo H-space.

Theorem 6. Let (X, q) be a pseudo H-space. The mapping $f : X \times X \rightarrow R$. Given any ì $\in$ R. Suppose that

(1') for each $y \in X$, for each nonempty finite subset A of $\{x \in X : f(x,y) > ì\}$, $q(\triangle^{|A|-1}) \subset \{x \in X : f(x,y) > ì\}$,

(2) for each for each nonempty finite subset A of X and $z \in \triangle^{|A|-1}$, $f(w,w) \leq ì$ for all $w \in q(z)$.

Then for any given nonempty finite subset L of X, there is a $y_0 \in X$ such that $f(x, y_0) \leq ì$ for all $x \in L$. In additional, if $\bigcap_{x \in M} \{y \in X : f(x,y) \leq ì\}$ is contained in a compact subset of X for some nonempty finite subset M of X, then there is a $y_0 \in X$ such that $f(x, y_0) \leq ì$ for all $x \in X$.

We note that under the hypothesis of Theorem 6, if ì $= \sup_{x \in X} f(x,x)$, then the last assertion can deduce that the following inequality hold :

$$\inf_{y \in X} \sup_{x \in X} f(x,y) \leq \sup_{x \in X} f(x,x).$$

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行政院國家科學委員會補助專題研究計畫成果報告

集合值映射在積擬H-空間中之定點定理的探討及其應用

The study of the fixed point theorems of set-valued mappings on the product pseudo H-spaces and its applications

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